MATRIX

Matrix:- An ordered rectangular array of mn numbers having m horizontal lines (rows) & n vertical lines (columns) is called a matrix of order m by n and is written as $m \times n$.

Such an arrangement is enclosed in a bracket [ ] or parenthesis ( ).

eg:
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{m \times n}$$

← Rows (m)

↑ Columns (n)

↳ order of the matrix

⇒ **General form of a Matrix-**

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & & a_{mn} \end{bmatrix}$$

→ **Order of a Matrix-**

$m \times n$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3 \times 2}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}$$

Construction of a matrix

Construct a matrix A of order 3×2 whose elements are given by –

$a_{ij} = 1 + 3i - 2j$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = A = \begin{bmatrix} 2 & 0 \\ 5 & 3 \\ 8 & 6 \end{bmatrix}$$

$a_{11} \Rightarrow 1 + (3*1) - (2\times1)$
$= 4 - 2$
$= 2$

$a_{12} = 1 + (3*1) - (2\times2)$
$= 4 - 4$
$= 0$

$a_{21} = 1 + (3\times2) - (2\times1)$
$= 7 - 2$
$= 5$

$a_{22} = 1 + (3\times2) - (2\times2)$
$= 7 - 4$
$= 3$

$a_{31} = 1 + (3\times3) - (2*1)$
$= 10 - 2$
$= 8$

$a_{32} = 1 + (3\times3) - (2\times2)$
$= 10 - 4$
$= 6$

→ Types of matrices:- one row & any no. of columns.

① Row Matrix:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

② Column matrix:- one column & any no. of rows.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{1\times 3} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

③ Null matrix :- whose all elements are zero (0).

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

④ Square matrix :- when no. of rows is equal to the no. of columns.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$$

3→

⑤ Identity matrix / unit matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Principal diagonal - only (1)

3→

⑥ Diagonal matrix :-

$$\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Principal diagonal => any number.

⑦ Scaler matrix :- principal diagonal => same

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

8) Upper Triangular Matrix:-

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

9) Lower Triangular Matrix:-

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

10) Comparable matrices :- When rows & columns of both matrices are equal.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times2} \qquad B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}_{3\times2}$$

Order should be same !

11) Equality of matrices -

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \qquad B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}$$

$$A = B$$

$a = 1$, $b = 2$, $c = 3$, $d = 4$.

$$\begin{bmatrix} 7 & 3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} a & 3 \\ c & 3 \end{bmatrix}$$

$a = 7$

$c = 2$

12) Sub-matrix :-

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right] \leftarrow \qquad \begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}$$

Operation on Matrices:-

i) Addition
ii) Subtraction
iii) Scalar multiplication
iv) Multiplication of matrices.

Order should be same

i) Addition:-

Corresponding elements

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 0 & 2 \end{bmatrix}$$

→ find $A+B$ -

$$A+B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 0 & 2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 6 & 8 \\ 3 & 6 \end{bmatrix}$$

ii) Subtraction:-

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 0 & 2 \end{bmatrix}$$

→ find $A-B$ -

$$A-B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 0 & 2 \end{bmatrix}$$

$$\underline{A-B} = \begin{bmatrix} -4 & -4 \\ 3 & 2 \end{bmatrix}$$

(ii) Scalar multiplication:-

① $A = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 4 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

find $Ax =$

$$Ax = \begin{bmatrix} 4x & 5x & 6x \\ 7x & 4x & 2x \\ x & x & 0 \end{bmatrix}$$

② $B = \begin{bmatrix} 2 & 4 \\ 5 & 8 \end{bmatrix}$, $2B$

$$2B = \begin{bmatrix} 4 & 8 \\ 10 & 16 \end{bmatrix}$$

Q:- if $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 3 & 2 \end{bmatrix}$

then find $3A + 2B$.

Sol $3A = \begin{bmatrix} 3 & 6 & 12 \\ 9 & 0 & 15 \end{bmatrix}$, $2B = \begin{bmatrix} 4 & 8 & 2 \\ 0 & 6 & 4 \end{bmatrix}$

$$3A+2B = \begin{bmatrix} 3 & 6 & 12 \\ 9 & 0 & 15 \end{bmatrix} + \begin{bmatrix} 4 & 8 & 2 \\ 0 & 6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 14 & 14 \\ 9 & 6 & 19 \end{bmatrix}$$

Q=> find the values of a, b, c & d from the matrix equation.

$$2\begin{bmatrix} a & c \\ b & d \end{bmatrix} + 5\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 0 & -5 \\ 4 & 6 \end{bmatrix}$$

Sol=>

$$\begin{bmatrix} 2a & 2c \\ 2b & 2d \end{bmatrix} + \begin{bmatrix} 5 & -5 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 0 & -15 \\ 12 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 2a & 2c \\ 2b & 2d \end{bmatrix} = \begin{bmatrix} 0 & -15 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 5 & -5 \\ 0 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 2a & 2c \\ 2b & 2d \end{bmatrix} = \begin{bmatrix} -5 & -10 \\ 12 & 8 \end{bmatrix}$$

$\therefore 2a = 5, 2b = 12, 2c = -10, 2d = 8$

By Equality of matrices we get

$a = -5/2, b = 12/2 = 6, c = -10/2 = -5, d = 8/2 = 4$

$a = -5/2$ $\quad b = 6$

$c = -5$ $\quad d = 4$ ≠

Q=> if $A = \begin{bmatrix} 6 & 2 \\ 2 & 4 \\ 0 & -3 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 4 \\ 1 & -3 \\ 2 & 1 \end{bmatrix}$

then find the matrix C such that A + B + C is a Zero matrix.

Sol=> $A+B+C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 2 \\ 2 & 4 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 1 & -3 \\ 2 & 1 \end{bmatrix} + C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 6 \\ 3 & 1 \\ 2 & -2 \end{bmatrix} + C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 6 \\ 3 & 1 \\ 2 & -2 \end{bmatrix}$$

$$C = \begin{bmatrix} -6 & -6 \\ -3 & -1 \\ -2 & 2 \end{bmatrix}$$

Q→ find the matrix X such that $= 2X - \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 0 & 5 \end{bmatrix}$

Sol→ $2X = \begin{bmatrix} 2 & 6 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$

$$2X = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{4}{2} & \frac{3}{2} \\ \frac{5}{2} & \frac{6}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & \frac{3}{2} \\ \frac{5}{2} & 3 \end{bmatrix}$$

$=$

Q. Determine the matrices A & B such that

$$A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \quad \text{--- (I)} \quad \text{and}$$

$$2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{--- (II)}$$

Sol: Multiplying eq (I) by (2)

$$2A + 4B = \begin{bmatrix} 2 & 4 & 0 \\ 12 & -6 & 6 \\ -10 & 6 & 2 \end{bmatrix} \quad \text{--- (III)}$$

Subtracting eq (II) from (III).

$$\begin{array}{r} 2A - B \\ 2A + 4B \\ - \quad - \\ \hline -5B \end{array}$$

$$(2A - B) - (2A + 4B) = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 0 \\ 12 & -6 & 6 \\ -10 & 6 & 2 \end{bmatrix}$$

$$-5B = \begin{bmatrix} 0 & -5 & 5 \\ -10 & 5 & 0 \\ 10 & -5 & 0 \end{bmatrix}$$

$$B = \frac{-1}{5} \begin{bmatrix} 0 & -5 & 5 \\ -10 & 5 & 0 \\ 10 & -5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$$

Putting the value of B in eq. (I)

$$A + 2\begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -2 \\ 4 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$$

$$A + \begin{bmatrix} 0 & 2 & -2 \\ 4 & -2 & 0 \\ -4 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2 & -2 \\ 4 & -2 & 0 \\ -4 & 2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

Q=> if $A = \begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 1 \\ 6 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & -3 \\ 3 & 2 & -1 \end{bmatrix}$, $C \begin{bmatrix} -1 & 1 & -1 \\ -2 & 2 & 2 \\ -1 & -3 & -2 \end{bmatrix}$

then find = $\frac{1}{3}A + \frac{2}{5}B - 4C$

Sol=> $\frac{1}{3}A + \frac{2}{5}B - 4C$

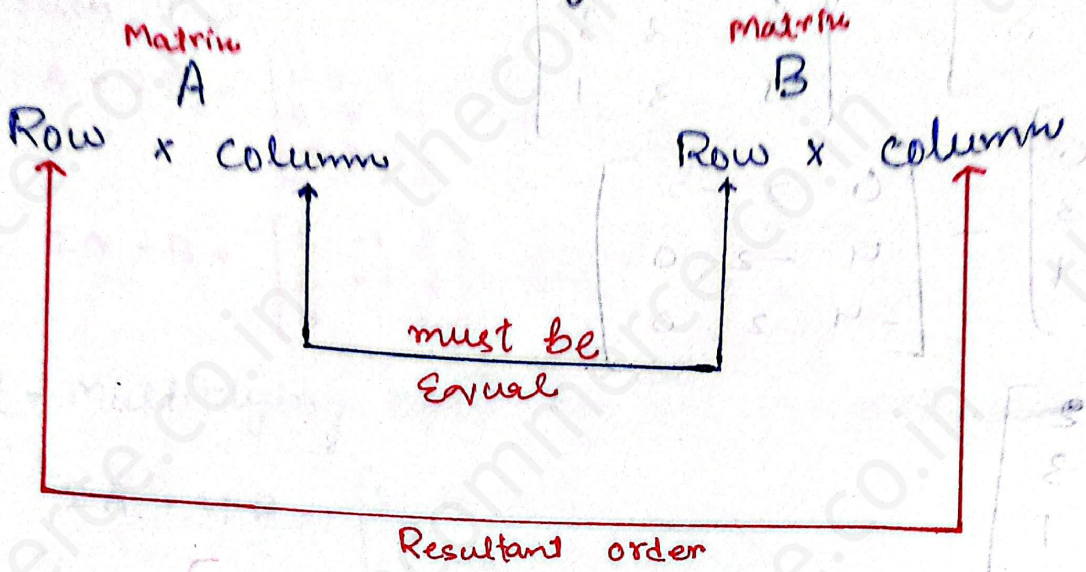
$$\frac{1}{3}\begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 1 \\ 6 & 1 & 2 \end{bmatrix} + \frac{2}{5}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & -3 \\ 3 & 2 & -1 \end{bmatrix} - 4\begin{bmatrix} -1 & 1 & -1 \\ -2 & 2 & 2 \\ -1 & -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{5}{3} & 0 & \frac{1}{3} \\ 2 & \frac{1}{3} & \frac{2}{3} \end{bmatrix} + \begin{bmatrix} \frac{2}{5} & -\frac{2}{5} & \frac{2}{5} \\ \frac{4}{5} & 0 & -\frac{6}{5} \\ \frac{6}{5} & \frac{4}{5} & -\frac{2}{5} \end{bmatrix} - \begin{bmatrix} -4 & 4 & -4 \\ -8 & 8 & 8 \\ -4 & -12 & -8 \end{bmatrix}$$

$$\begin{bmatrix} \frac{26}{15} & \frac{-11}{15} & \frac{16}{15} \\ \frac{37}{15} & 0 & \frac{-13}{15} \\ \frac{16}{5} & \frac{17}{15} & \frac{4}{15} \end{bmatrix} - \begin{bmatrix} -4 & 4 & -4 \\ -8 & 8 & 8 \\ -4 & -12 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{86}{15} & \frac{-71}{15} & \frac{76}{15} \\ \frac{157}{15} & -8 & \frac{-133}{15} \\ \frac{36}{5} & \frac{197}{15} & \frac{124}{15} \end{bmatrix}$$

↳ Multiplication of Matrices :-



* Columns of 'A' must be equal to Rows of 'B'.

Eg: Let $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}_{2\times2}$ and $B = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}_{2\times2}$

find AB.

$\vec{A} \cdot \downarrow B$

Sol:-

$$AB = \begin{bmatrix} (1\times1)+(-1\times4) & (1\times1)+(-1\times-1) \\ (2\times1)+(-1\times4) & (2\times1)+(-1\times-1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+(-4) & 1+1 \\ 2+(-4) & 2+1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 2 \\ -2 & 3 \end{bmatrix}$$

Q: $A = \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$ & $B \begin{bmatrix} 5 & 2 & 1 \\ 4 & 3 & 0 \end{bmatrix}$, then find AB.

Q. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = 0$ where I is identity matrix of order 3 & 0 is a null Matrix of order 3.

Sol. $A^3 - 23A - 40I = 0$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+12 & 2+(-4)+6 & 3+2+3 \\ 3+(-6)+4 & 6+4+2 & 9+(-2)+1 \\ 4+6+4 & 8+(-4)+2 & 12+2+1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 19+12+32 & 38+(-8)+16 & 57+4+8 \\ 1+36+32 & 2+(-24)+16 & 3+12+8 \\ 14+18+60 & 28+(-12)+30 & 42+6+15 \end{bmatrix} - \begin{bmatrix} -17 & 46 & 69 \\ 69 & -86 & 23 \\ 92 & 46 & -17 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} -17 & 46 & 69 \\ 69 & -86 & 23 \\ 92 & 46 & -17 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now Solve →

$A^3 - 23A - 40I = 0$

→ $A^3 - 23A - 40I$

$$\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23 \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46-0 & 69-69-0 \\ 69-69-0 & -6+46-40 & 23-23-0 \\ 92-92-0 & 46-46-0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence prove- $A^3 - 23A - 40I = O$

Q. If $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ 3 & 5 \end{bmatrix}$, then find the values of a & b such that $AB = BA$.

Sol:- $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3a+6 & 3b+10 \\ 4a+3 & 4b+5 \end{bmatrix} = \begin{bmatrix} 3a+4b & 2a+b \\ 29 & 11 \end{bmatrix}$

By equality of matrices.

$\Rightarrow 4a+3 = 29 \quad -(1)$

$4a = 29-3$

$4a = 26$

$a = \frac{26}{4_2} = \frac{13}{2}$

$4b+5 = 11 \quad -(11)$

$4b = 11-5$

$4b = 6$

$b = \frac{6^3}{4_2} = \frac{3}{2}$

$\therefore a = 13/2$ & $b = 3/2$ ≠

Determinants :-

↳ Determinant of a square matrix

→ of order 2

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \Rightarrow (a_{11} \times a_{22}) - (a_{12} \times a_{21})$

eg:- $A = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 4 \\ 3 & 9 \end{vmatrix}$.

$|A| = (5 \times 9) - (4 \times 3)$

$|A| = 45 - 12$

$|A| = 33$

Determinant of Matrix of Order 3:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$|A| = a_{11}\left[(a_{22}\times a_{33}) - (a_{23}\times a_{32})\right] - a_{12}\left[(a_{21}\times a_{33}) - (a_{23}\times a_{31})\right] + a_{13}\left[(a_{21}\times a_{32}) - (a_{22}\times a_{31})\right]$$

Q⇒ Evaluate. $\begin{bmatrix} 1 & -2 & 3 \\ 4 & -1 & -2 \\ -2 & 1 & 5 \end{bmatrix}$

Sol⇒ $|A| = \begin{vmatrix} 1 & -2 & 3 \\ 4 & -1 & -2 \\ -2 & 1 & 5 \end{vmatrix}$

$$|A| = 1\begin{vmatrix} -1 & -2 \\ 1 & 5 \end{vmatrix} + 2\begin{vmatrix} 4 & -2 \\ -2 & 5 \end{vmatrix} + 3\begin{vmatrix} 4 & -1 \\ -2 & 1 \end{vmatrix}$$

$$|A| = 1\left[(-1\times 5) - (-2\times 1)\right] + 2\left[(4\times 5) - (-2\times(-2))\right] + 3\left[(4\times 1) - (-1\times(-2))\right]$$

$$|A| = 1(-5+2) + 2(20-4) + 3(4-2)$$

$$|A| = 1(-3) + 2(16) + 3(2)$$

$$|A| = -3 + 32 + 6$$

$$|A| = \underline{35}$$

Minors and Co-factors of Elements of Determinants.

↳ **Minor :-** The Minor of any elements in the determinant of A is the determinant obtained by deleting the row + column in which the element lies.

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Every element has its own minor.

$M_{ij} = M_{11}, M_{12}, M_{13} - \text{——} M_{33}$

eg:

$a_{11} = M_{11} \Rightarrow \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$a_{12} = M_{12} \Rightarrow \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$a_{33} = M_{33} \Rightarrow \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Co-factors :- The co-factor of any element in the determinant of A is the Minor of that element in the det A with proper sign depending on the number of row & column in which the element lies.

→ Denote

$A_{ij} = (-1)^{i+j} \times M_{ij}$

$A_{21} = (-1)^{2+1} \times M_{21}$

$A_{21} = (-1)^{3} \times M_{21}$

$A_{21} = -1 \times M_{21}$

$A_{31} = (-1)^{3+1} \times M_{31}$

$A_{31} = (-1)^{4} \times M_{31}$

$A_{31} = 1 \times M_{31}$

Minors and Co-factors

M_{ij} A_{ij}

Q-> find the co-factors of the elements of

$$\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix}$$

Sol=> $A_{ij} = (-1)^{i+j} \times M_{ij}$

$A_{11} = (-1)^2 \times M_{11}$

$A_{11} = 1 \times 5 = 5$

$A_{12} = (-1)^{1+2} \times M_{12}$

$A_{12} = -1 \times 3$

$A_{12} = -3$

$A_{21} = (-1)^{2+1} \times M_{21}$

$A_{21} = -1 \times 2$

$A_{21} = -2$

$A_{22} = (-1)^{2+2} \times M_{22}$

$A_{22} = 1 \times 1$

$A_{22} = 1$

Q⇒ find the co-factors of the elements of

$$\begin{vmatrix} 2 & -3 & 7 \\ 4 & -9 & 1 \\ 6 & 5 & 0 \end{vmatrix}$$

Sol⇒ $A_{ij} = (-1)^{i+j} \times M_{ij}$

$A_{11} = (-1)^{1+1} \times M_{11}$

$A_{11} = 1 \times \begin{vmatrix} -9 & 1 \\ 5 & 0 \end{vmatrix}$

$A_{11} = 1(-5)$

$A_{11} = 5$

$A_{12} = (-1)^{1+2} \times M_{12}$

$A_{12} = -1 \times \begin{vmatrix} 4 & 1 \\ 6 & 0 \end{vmatrix}$

$A_{12} = -1(-6)$

$A_{12} = 6$

$A_{13} = (-1)^{1+3} \times M_{13}$

$A_{13} = 1 \begin{vmatrix} 4 & -9 \\ 6 & 5 \end{vmatrix}$

$A_{13} = 1(20+54)$

$A_{13} = 1(74)$

$A_{13} = 74$

$A_{21} = (-1)^{2+1} \times M_{21}$

$A_{21} = -1 \begin{vmatrix} -3 & 7 \\ 5 & 0 \end{vmatrix}$

$A_{21} = -1(-35)$

$A_{21} = 35$

$A_{22} = (-1)^{2+2} \times M_{22}$

$A_{22} = 1 \cdot \begin{vmatrix} 2 & 7 \\ 6 & 0 \end{vmatrix}$

$A_{22} = 1(-42)$

$A_{22} = -42$

$A_{23} = (-1)^{2+3} \times M_{23}$

$A_{23} = -1 \times \begin{vmatrix} 2 & -3 \\ 6 & 5 \end{vmatrix}$

$A_{23} = -1(10+18)$

$A_{23} = -1(28)$

$A_{23} = -28$

$A_{31} = (-1)^{3+1} \times M_{31}$

$A_{31} = 1 \cdot \begin{vmatrix} -3 & 7 \\ -9 & 1 \end{vmatrix}$

$A_{31} = 1(-3+63)$

$A_{31} = 1(60)$

$A_{31} = 60$

$A_{32} = (-1)^{3+2} \times M_{32}$

$A_{32} = -1 \times \begin{vmatrix} 2 & 7 \\ 4 & 1 \end{vmatrix}$

$A_{32} = -1(2-28)$

$A_{32} = -1(-26)$

$A_{32} = 26$

$A_{33} = (-1)^{3+3} \times M_{33}$

$A_{33} = 1 \cdot \begin{vmatrix} 2 & -3 \\ 4 & -9 \end{vmatrix}$

$A_{33} = 1(-18+12)$

$A_{33} = 1(-6)$

$A_{33} = -6$

$A_{11} = -5$

$A_{12} = 6$

$A_{13} = 74$

$A_{21} = 35$

$A_{22} = -42$

$A_{23} = -28$

$A_{31} = 60$

$A_{32} = 26$

$A_{33} = -6$

Solution of System of linear equations in three unknown values.

→ By Cramer's Rule:

Q-> Solve the following system of equations by cramer's rule -

$$x + y + z = 3$$
$$2x + 3y + 4z = 19$$
$$x + 2y - 4z = -1$$

Sol=>

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{bmatrix}$$

$$|A| = 1(12-8) - 1(-8-4) + 1(4-3)$$
$$|A| = 1(-20) - 1(-12) + 1(1)$$
$$|A| = -20 + 12 + 1$$
$$|A| = -7 \neq 0$$

$$x = \frac{|A_1|}{|A|}, \quad y = \frac{|A_2|}{|A|}, \quad z = \frac{|A_3|}{|A|}$$

$$|A_1| = \begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 4 \\ -12 & 2 & -4 \end{vmatrix}$$

$$|A_1| = 3\begin{vmatrix} 3 & 4 \\ 2 & -4 \end{vmatrix} - 1\begin{vmatrix} 9 & 4 \\ -1 & -4 \end{vmatrix} + 1\begin{vmatrix} 9 & 3 \\ -1 & 2 \end{vmatrix}$$

$$|A_1| = 3(-12-8) - 1(-36+4) + 1(18+3)$$
$$|A_1| = 3(-20) - 1(-32) + 1(21)$$
$$|A_1| = -60 + 32 + 21 \Rightarrow \boxed{|A_1| = -7}$$

for $|A_2| = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 9 & 4 \\ 1 & -1 & -4 \end{bmatrix}$

$|A_2| = 1(-36+4) - 3(-8-4) + 1(-2-9)$

$|A_2| = 1(-32) - 3(-12) + 1(-11)$

$|A_2| = -32 + 36 - 11$

$|A_2| = -7$

for $|A_3| = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 9 \\ 1 & 2 & -1 \end{bmatrix}$

$|A_3| = 1(-3-18) - 1(-2-9) + 3(4-3)$

$= 1(-21) - 1(-11) + 3(1)$

$= -21 + 11 + 3$

$= -7$

Now $x = \frac{|A_1|}{|A|} = \frac{-7}{-7} = 1$

$y = \frac{|A_2|}{|A|} = \frac{-7}{-7} = 1$

$z = \frac{|A_3|}{|A|} = \frac{-7}{-7} = 1$

$\therefore x = 1, \; y = 1 \; \& \; z = 1.$

Transpose of a Matrix:-

A matrix obtained by interchanging the rows & columns is called the transpose of that matrix.

It is denoted by :- A^T or A'

eg:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 5 & 2 \\ 2 & 6 & 3 \\ 3 & 7 & 4 \\ 4 & 8 & 6 \end{bmatrix}$$

Properties of a transpose of a Matrix:-

1. $(A^T)^T = A$
2. $(A+B)^T = A^T + B^T$
3. $(AB)^T = B^T \times A^T$

1. $A^T = \begin{bmatrix} 1 & 5 & 2 \\ 2 & 6 & 3 \\ 3 & 7 & 4 \\ 4 & 8 & 6 \end{bmatrix}$ $(A^T)^T = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 6 \end{bmatrix} = A$

2. $(A+B)^T = A^T + B^T$

$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 2 \\ 6 & 5 & 1 \end{bmatrix}$

$(A+B) = \begin{bmatrix} 3 & 5 & 10 \\ 5 & 7 & 10 \\ 9 & 11 & 10 \end{bmatrix}$

$(A+B)^T = \begin{bmatrix} 3 & 5 & 9 \\ 5 & 7 & 11 \\ 10 & 10 & 10 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 6 \\ 1 & 2 & 5 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 9 \\ 5 & 7 & 11 \\ 10 & 10 & 10 \end{bmatrix}$

(3) $(AB)^T = B^T \cdot A^T$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}_{2\times3}, \quad B = \begin{bmatrix} 0 & 2 \\ 4 & 3 \\ 1 & 5 \end{bmatrix}_{3\times2}$$

$$(AB)^T = \begin{bmatrix} 11 & 23 \\ 7 & 22 \end{bmatrix}_{2\times2}$$

$$(AB)^T = \begin{bmatrix} 11 & 7 \\ 23 & 22 \end{bmatrix}$$

Now $B^T \cdot A^T$

$$B^T \cdot A^T = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 3 & 5 \end{bmatrix}_{2\times3} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix}_{3\times2}$$

$$B^T \cdot A^T = \begin{bmatrix} 11 & 7 \\ 23 & 22 \end{bmatrix}$$

Symmetric & Skew-Symmetric Matrices

Symmetric Matrix:- If the transpose of a matrix is equal to the matrix itself, then it is called a symmetric matrix.

i.e. $A^T = A$

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 7 \\ 5 & 7 & 4 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 7 \\ 5 & 7 & 4 \end{bmatrix}$$

Skew symmetric Matrix:- +ive became -ive & vice versa

$$A = -A^T$$

$$A = \begin{bmatrix} 0 & 2 & 5 \\ -2 & 0 & 3 \\ -5 & -3 & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & -2 & -5 \\ 2 & 0 & -3 \\ 5 & 3 & 0 \end{bmatrix}$$

* Only square matrices can be symmetric & skew-symmetric.

Q-> if $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 5 & 4 & 5 \end{bmatrix}$, then show that- $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Sol=>

$$\begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 5 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 5 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 5 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix} \Leftarrow A^T$$

$$(A + A^T) = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 5 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 5 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$(A + A^T) = \begin{bmatrix} 6 & 6 & 8 \\ 6 & 10 & 7 \\ 8 & 7 & 10 \end{bmatrix}$$

$$\frac{1}{2}(A+A^T) = \begin{bmatrix} 3 & 3 & 4 \\ 3 & 5 & 7/2 \\ 4 & 7/2 & 5 \end{bmatrix}$$

$$(A-A^T) = \begin{bmatrix} 0 & -2 & -2 \\ 2 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A-A^T) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T) =$$

$$= \begin{bmatrix} 3 & 3 & 4 \\ 3 & 5 & 7/2 \\ 4 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 5 & 4 & 5 \end{bmatrix}$$

Adjoint of a Matrix :-

1. $A_{ij} = (-1)^{i+j} \times M_{ij}$
2. Matrix of co-factors
3. Transpose ↑

Matrix of order 2.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \Rightarrow \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} \Leftarrow \text{adj of } A$$

adj A

Adj of order 2 Shortcut method:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad \text{adj} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$A_{11} = (-1)^{1+1} \times M_{11}$

$A_{11} = 1 \times 4$

$A_{11} = 4$

$A_{12} = (-1)^{1+2} \times M_{12}$

$A_{12} = -1 \times 3$

$A_{12} = -3$

$A_{21} = (-1)^{2+1} \times M_{21}$

$A_{21} = -1 \times 2$

$A_{21} = -2$

$A_{22} = (-1)^{2+2} \times M_{22}$

$A_{22} = 1 \times 1$

$A_{22} = 1$

$\therefore A = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} = A^T \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Matrix of co-factors → adj A ↑

Adjoint of Square Matrix of Order 3 ×

Co-factors → Matrix of cofactors:-

↓

Transpose = adj A

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$A_{11} = (-1)^{1+1} \times M_{11}$

$= 1 \times \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$

$= 1(2-3)$

$= 1(-1)$

$\underline{A_{11} = -1}$

$A_{12} = (-1)^{1+2} \times M_{12}$

$A_{12} = -1(1-9)$

$A_{12} = -1(-8)$

$A_{12} = 8$

$A_{13} = (-1)^{1+3} \cdot M_{13}$

$A_{13} = 1(1-6)$

$A_{13} = 1(-5)$

$A_{13} = -5$

$A_{21} = (-1)^{2+1} \cdot M_{21}$

$= -1 \cdot (1-2)$

$= -1(-1)$

$= 1$

$A_{22} = (-1)^{2+2} \cdot M_{22}$

$= 1(0-6)$

$= 1(-6)$

$= -6$

$A_{23} = (-1)^{2+3} \cdot M_{23}$

$= -1(0-3)$

$= -1(-3)$

$= 3$

$A_{31} = (-1)^{3+1} \cdot M_{31}$

$= 1(3-4)$

$= 1(-1)$

$= -1$

$A_{32} = (-1)^{3+2} \cdot M_{32}$

$= -1(0-2)$

$= -1(-2)$

$= 2$

$A_{33} = (-1)^{3+3} \cdot M_{33}$

$= 1(0-1)$

$= 1(-1)$

$= -1$

Matrix of co-factor $= \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}$

$\text{adj } A = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Inverse of a Square matrix:

Steps involved in finding the inverse of a Matrix A →

i) find $|A|$. det A

ii) $|A| = 0$, Singular matrix, Inverse does not exists.

iii) $|A| \neq 0$, then find

iv) Adj A.

v) $A^{-1} = \frac{adj\ A}{|A|}$ or $\frac{1}{|A|} \times adj\ A$

Q: find the inverse of the matrix-

$$A = \begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix}$$

Sol:- find $|A|$

$$|A| = \begin{vmatrix} 2 & 5 \\ 3 & -7 \end{vmatrix}$$

$$|A| = (-14 - 15)$$

$$|A| = (-29) \neq 0$$

$$adj\ A = \begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} -7 & -5 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$A^{-1} = \begin{bmatrix} \frac{-7}{-29} & \frac{-5}{-29} \\ \frac{-3}{-29} & \frac{2}{-29} \end{bmatrix}$$

Or $A^{-1} = \frac{1}{|A|} \times adj\ A$

$$= \frac{1}{-29} \cdot \begin{bmatrix} -7 & -5 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-7}{-29} & \frac{-5}{-29} \\ \frac{-3}{-29} & \frac{2}{-29} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{7}{29} & \frac{5}{29} \\ \frac{3}{29} & \frac{-2}{29} \end{bmatrix}$$

Q⇒ $A=\begin{bmatrix}1 & 3 & 2\\ 1 & 1 & 1\\ 2 & -3 & -1\end{bmatrix}$ – find inverse.

Sol⇒ $|A| = 1(-1+3) - 3(-1-2) + 2(-3-2)$

$|A| = 1(2) - 3(-3) + 2(-5)$

$|A| = 2+9-10$

$|A| = 1 \neq 0$

$\text{adj } A = \begin{bmatrix}1 & 3 & 2\\ 1 & 1 & 1\\ 2 & -3 & -1\end{bmatrix}$

$A_{11} = (-1)^{1+1} \cdot M_{11}$

$A_{11} = 1 \cdot \begin{vmatrix}1 & 1\\ -3 & -1\end{vmatrix}$

$A_{11} = 1(-1+3)$

$A_{11} = 2$

$A_{12} = (-1)^{1+2} \cdot M_{12}$

$A_{12} = -1 \cdot (-1-2)$

$A_{12} = -1(-3)$

$A_{12} = 3$

$A_{13} = (-1)^{1+3} \cdot M_{13}$

$A_{13} = 1(-3-2)$

$A_{13} = 1(-5)$

$A_{13} = -5$

$A_{21} = (-1)^{2+1} \cdot M_{21}$

$A_{21} = -1(-3+6)$

$A_{21} = -1(3)$

$A_{21} = -3$

$A_{22} = (-1)^{2+2} \cdot M_{22}$

$A_{22} = 1(-1-4)$

$A_{22} = 1(-5)$

$A_{22} = -5$

$A_{23} = (-1)^{2+3} \cdot M_{23}$

$A_{23} = -1(-3-6)$

$A_{23} = -1(-9)$

$A_{23} = 9$

$A_{31} = (-1)^{3+1} \cdot M_{31}$

$A_{31} = 1(3-2)$

$A_{31} = 1(1)$

$A_{31} = 1$

$A_{32} = (-1)^{3+2} \cdot M_{32}$

$A_{32} = -1(1-2)$

$A_{32} = -1(-1)$

$A_{32} = 1$

$A_{33} = (-1)^{3+3} \cdot M_{33}$

$A_{33} = 1(1-3)$

$A_{33} = 1(-2)$

$A_{33} = -2$

adjoint = matrix of co-factor $\Rightarrow \begin{bmatrix} 2 & 3 & -5 \\ -3 & -5 & 9 \\ 1 & 1 & -2 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & -5 & 1 \\ -5 & 9 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{1} & \frac{-3}{1} & \frac{1}{1} \\ \frac{3}{1} & \frac{-5}{1} & \frac{1}{1} \\ \frac{-5}{1} & \frac{9}{1} & \frac{-2}{1} \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -3 & 1 \\ 3 & -5 & 1 \\ -5 & 9 & -2 \end{bmatrix}$$

To check:

$A \times A^{-1} = I \rightarrow$ unit matrix.

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 2 & -3 & 1 \\ 3 & -5 & 1 \\ -5 & 9 & -2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2+9-10$

$-3-15+18$

$-18+18$

$= 0$

$1+3-4$

$2+3-5$

$-3-5+9$

$-8+9$

$1+1-2$

$4-9+5$

$-5+5$

$-6+15-9$

$-6+15+(-9)$

$= 0$

$2-3+2$

$-1+2$

Solution of a system of linear equations:

↳ By Matrix method:

$$a_{11}\cdot x + a_{12}\cdot y + a_{13}\cdot z = b_1$$
$$a_{21}\cdot x + a_{22}\cdot y + a_{23}\cdot z = b_2$$
$$a_{31}\cdot x + a_{32}\cdot y + a_{33}\cdot z = b_3$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$AX = B \quad \text{—①}$$

find A^{-1}.

Multiply A^{-1} b/sides of the eq.

$$A^{-1}\cdot AX = A^{-1}\cdot B$$
$$I\,X = A^{-1}\cdot B$$
$$\underline{X = A^{-1}\cdot B}$$

Q-> Solve the following system of equations by matrix method.

$$x + y + z = 3$$
$$2x + 3y + 4z = 9$$
$$x + 2y - 4z = -1$$

Sol=>

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 9 \\ -1 \end{bmatrix}$$

$$AX = B$$

where $X = A^{-1}B$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{vmatrix}$$

$$|A| = 1(-12-8) - 1(-8-4) + 1(4-3)$$

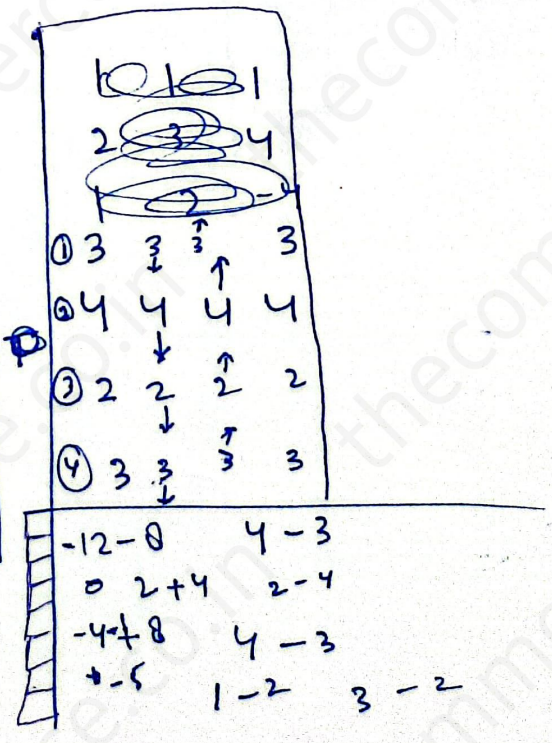
$$|A| = 1(-20) - 1(-12) + 1(1)$$

$$|A| = -20 + 12 + 1$$

$|A| = -7 \neq 0$ A^{-1} will exist

for $\text{adj } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 3 & \begin{bmatrix} 2 \\ -4 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix} & 3 \\ 4 & & & 4 \\ 2 & 1 & 1 & 2 \\ 3 & 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -20 & 6 & 1 \\ 12 & -5 & -2 \\ 1 & -1 & 1 \end{bmatrix}$$



$$\text{adj } A = \begin{bmatrix} -20 & 6 & 1 \\ 12 & -5 & -2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = \begin{bmatrix} \frac{-20}{-7} & \frac{6}{-7} & \frac{1}{-7} \\ \frac{12}{-7} & \frac{-5}{-7} & \frac{-2}{-7} \\ \frac{1}{-7} & \frac{-1}{-7} & \frac{1}{-7} \end{bmatrix} \Longrightarrow \begin{bmatrix} \frac{20}{7} & \frac{-6}{7} & \frac{-1}{7} \\ \frac{-12}{7} & \frac{5}{7} & \frac{2}{7} \\ \frac{1}{7} & \frac{1}{7} & \frac{-1}{7} \end{bmatrix}$$

$A^{-1} \cdot AX = A^{-1} \cdot B$

We know that $A^{-1} \cdot A = I$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{20}{7} & -\frac{6}{7} & -\frac{1}{7} \\ -\frac{12}{7} & \frac{5}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ -1 \end{bmatrix}$$

$$\frac{20}{7} \times \frac{3}{1} = \frac{60}{7}$$

$$-\frac{54}{7}$$

$$\frac{1}{7}$$

$$\frac{60-54+1}{7} = 7$$

$$\frac{-36+45-2}{7}$$

$$\frac{-349+1}{7}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

By the equality of two matrices.

$\therefore x = 1, y = 1, z = 1$

Q- find A^{-1}, and hence solve the system of equations -

$$x + 2y + 5z = 10$$
$$x - y - z = -2$$
$$2x + 3y - z = -11$$

Sol

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{adj\,A}{|A|}$$

$$|A| = \begin{vmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{vmatrix}$$

$$|A| = 1(1+3) - 2(-1+2) + 5(3+2)$$

$$|A| = 1(4) - 2(1) + 5(5)$$

$$|A| = 4 - 2 + 25$$

$|A| = 27 \neq 0$ A^{-1} will exist

for $adj\,A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

$$adj\,A = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$$\begin{matrix} -1 & 3 & 2 & -1 \\ -1 & -1 & 5 & -1 \\ 1 & 2 & 1 & 1 \\ -1 & 3 & 2 & -1 \end{matrix}$$

$1+3 = 4$
$15+2 = 17$
$-2+5 = 3$
$-2+1 = -1$
$-1-10 = -11$
$5+1 = 6$
~~$3+2 = 5$~~
$4-3$
$-1-2$

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

$$A^{-1} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{3}{27} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27} \\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{1}{9} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9} \\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9} \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{1}{9} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9} \\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9} \end{bmatrix} \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

By equality of matrices.

$\therefore x = -1$

$y = -2$

$z = 3$

$\frac{40+(-34)}{27}6 + \frac{-11}{9} = \frac{54-297}{243}$

$\frac{-243}{243} \; {}^{-1}$

-1

$\frac{-10+22}{27} = \frac{12}{27} + \frac{-22}{9} = \frac{108-594}{243}$

$\frac{-486}{243} \; {}^{-2}$

$\frac{50-2}{27}\left[\frac{48}{27} + \frac{11}{9}\right] = \frac{432+297}{243}$

$\frac{729}{243} \; 3$

Q⇒ if $AB = \begin{bmatrix} 22 & 6 \\ 11 & 3 \end{bmatrix}$ and $A = \begin{bmatrix} 4 & 1 \\ 7 & 4 \end{bmatrix}$ then find the matrix B

Sol⇒ $A^{-1} = \frac{adjA}{|A|}$

$|A| = 16 - 7$

$|A| = 9 \neq 0$ A^{-1} exists

for $adjA = \begin{bmatrix} 4 & 1 \\ 7 & 4 \end{bmatrix}$

$adjA = \begin{bmatrix} 4 & -1 \\ -7 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 4/9 & -1/9 \\ -7/9 & 4/9 \end{bmatrix}$

$A^{-1} \cdot AB = B$

$I B = B$

$AB = \begin{bmatrix} 22 & 6 \\ 11 & 3 \end{bmatrix}$

$A^{-1} \cdot AB = \begin{bmatrix} 4/9 & -1/9 \\ -7/9 & 4/9 \end{bmatrix} \begin{bmatrix} 22 & 6 \\ 11 & 3 \end{bmatrix}$

$$B = \begin{bmatrix} \frac{77}{9} & \frac{7}{3} \\ \frac{-110}{9} & \frac{10}{3} \end{bmatrix}$$

$\frac{88-11}{9} = \frac{77}{9}$ $\quad \frac{24 \div 3}{9} = \frac{7}{3}$

$\frac{-154+44}{9} = \frac{-110}{9}$ $\quad \frac{7}{3}$

$\frac{-42+12}{9} = \frac{-30}{9} \quad \frac{10}{3}$